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Technical Notes

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Space–Time Analogy of Self-Similar Intense Vortices

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I. Introduction

THE present theoretical study aims to show the space–time analogy shared by a group of self-similar structure vortices. Based on order-of-magnitude arguments, we reduced the governing equations into the general form that has produced most of the previous classical vortex models. The system of equations is further reduced using a single independent variable that combined space and time. The governing equations of vortex decay have shown to be analogous to the steady-state subset. Therefore, if the time-independent solution is known, then the corresponding decaying version can be obtained through a straightforward variable transformation. Conversely, if the decaying solution is available, then the steady companion can also be recovered. On formalization of the method, three different examples are given to validate the concept. First, we proceed with the extension of the steady-state *n* family and Sullivan's vortex into time decaying. Second, by the reversing of the process, the steady-state group that gives rise to the diffusing Bellamy-Knights set of vortices is also recovered. Hence, the method provides steady solutions for Sullivan-like vortices rather than the original Sullivan's vortex.

The theoretical foundation of the present work can be traced back to the main rationale that has produced the celebrated vortices of Rankine,[1] Oseen[2]–Hamel,[3] Lamb,[4] Taylor,[5] Burgers,[6] and Sullivan.[7] The most well-known steady vortex formulation of the group is due to Rankine.[1] The model assumes both the radial and axial velocity components to be zero. The latter produces a Cauchy-type differential equation for the azimuthal momentum with two possible solutions for the tangential velocity. The first suggests a linear distribution inside the core, whereas the second gives a hyperbolic variation outside. Comparison with the observations shows this approximation to overestimate the velocity near the core. The Burgers[6] approach provides a smooth transition of the velocity from forced- to free-vortex modes and is in good agreement with the experiments. Vatistas et al.[8] proposed an alternative formulation that gives rise to a family of smooth velocity distributions having the Rankine–Helmholtz free and forced modes as asymptotes. The $n = 2$ member of the family gives a profile for the tangential velocity that is comparable to Burgers's, whereas the pressure is given explicitly. Based on Burgers's work, Sullivan[7] derived a two-celled steady vortex, characterized by a direction reversal of the radial and axial velocity components near the axis of rotation. Oseen[2]–Hamel[3] and Lamb[4] examined the decay of a potential vortex in a zero meridional field. In the beginning, the tangential velocity, vorticity, and the pressure at the origin are singular. As time evolves, the velocity begins to reduce, the vorticity diffuses outward, and the core dilates.

A solution in terms of a hypergeometric series was given by Kirde.[9] A decaying eddy with remarkable properties was derived by Taylor,[5] noting the similarity of vorticity diffusion to that of heat conduction. Rott[10] produced yet another noteworthy time-dependent solution by including the radial convective acceleration term in the tangential momentum equation. Bellamy-Knights[11] developed a two-cell diminishing vortex. A theoretical analysis concerning the time decay of laminar vortices due to the viscous effects has developed by Aboelkassem.[12] The subset of viscous vortices, which are diffusing in a zero meridional flow, has been thoroughly examined in the past. Viscous vortices decaying in a field in which both the radial and axial velocity components are present have not been equally explored. The present work contributes mainly to flows of the last kind. The principal motivation has always been the extension of the steady-state family of algebraic vortex equations proposed by Vatistas et al.[8] more than a decade ago. In the course of broadening the previous steady versions into the time-dependent type, we have formalized a space–time analogy and have elaborated on some unique properties applicable to all simple vortex flows of this type.

Vatistas et al.[13] recently laid out the fundamentals of the transformation. The methodology was tested using the $n = 2$ member of the *n* family of vortices for simplicity. Here we present the full transformation analysis that can be used to convert the steady vortices into the time decaying and vice versa and to validate the argument of space–time similarity shared by vortices of the intense kind.

II. Mathematical Formulation

Consider the motion of an incompressible, lamina, tubular, and intense vortex, where the tangential velocity is of a magnitude order higher than the radial and axial components. Because we are interested in time-dependent solutions to a particular class of vortices at which the steady-velocity vector has the standard general form,[6–8] let us, therefore, assume the velocity vector in our case varies with time as follows:

$$\boldsymbol{V}(r,t) = \langle V_r(r,t),\, V_\theta(r,t),\, V_z(r,t) = z h_z(r,t) \rangle$$

This particular flowfield is governed by the axisymmetric of continuity and Navier–Stokes equations in cylindrical coordinates and presented in dimensionless form as follows.

Continuity:

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + h_z = 0$$

$$\delta \qquad \delta \qquad \delta \tag{1}$$

Radial momentum:

$$\frac{\partial V_r}{\partial t} + Re\left(V_r \frac{\partial V_r}{\partial r} - \frac{V_\theta^2}{r}\right) = -Re\frac{\partial P}{\partial r} + \frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{V_r}{r^2}$$

$$\delta \qquad \frac{1}{\delta}(\delta\delta \qquad 1) \qquad \frac{1}{\delta} \qquad \delta \qquad \delta \qquad \delta \tag{2}$$

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Tangential momentum:

$$\frac{\partial V_\theta}{\partial t} + Re\left(V_r\frac{\partial V_\theta}{\partial r} + V_r\frac{V_\theta}{r}\right) = \frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r^2}$$

$$1 \qquad \frac{1}{\delta}(\delta 1 \qquad \delta 1) \qquad 1 \qquad 1 \qquad 1 \tag{3}$$

Axial momentum:

$$\frac{\partial h_z}{\partial t} + Re\left(V_r\frac{\partial h_z}{\partial r} + h_z^2\right) = -\frac{Re}{z}\frac{\partial P}{\partial z} + \frac{\partial^2 h_z}{\partial r^2} + \frac{1}{r}\frac{\partial h_z}{\partial r}$$

$$\delta \qquad \frac{1}{\delta}(\delta\delta \qquad \delta^2) \qquad \frac{1}{\delta} \qquad \delta \qquad \delta \tag{4}$$

where $t = \nu t^*/r_c^2, r = r^*/r_c, z = z^*/r_c, V_r(t,r) = V_r^*/V_{\theta c}, V_\theta(t,r) = V_\theta^*/V_{\theta c},\ V_z(t,r) = V_z^*/V_{\theta c} = z\,h_z(t,r),\ r_c$ is the core size, $V_{\theta c} = \Gamma_c/2\pi r_c$, and Γ_c is the circulation across the vortex center. The subscript c denotes the value of the core parameters (where the core is defined as the radius where the tangential velocity attains its maximum value), $P(t,r) = p/\rho V_{\theta_c}^2$ and $Re = V_{\theta_c} r_c/\nu$ is the vortex Reynolds number. Because we are dealing with strong vortices, the traditional assumption requiring that Vr and $h_z \ll V_\theta$, is implemented. In terms of order of magnitude, if V_θ is of $\mathcal{O}(1)$, then Vr and h_z are of $\mathcal{O}(\delta)$, where $\delta \sim 1/Re$. See Table 1 for typical δ values.

Based on order of magnitude arguments, one can bring the preceding system of equations into a simpler form. All terms in Eqs. (1) and (3) are of the same scale, and therefore, both equations should remain unchanged. Neglecting the terms in Eq. (2) of order δ, we obtain radial momentum

$$\frac{V_\theta^2}{r} = \frac{\partial P}{\partial r} \tag{5}$$

The axial momentum suggests that the static pressure should not vary appreciably in the z direction; therefore,

$$Re\frac{\partial P}{\partial z} \sim \delta \qquad \text{or} \qquad \frac{\partial P}{\partial z} \sim \delta^2, \qquad \text{that is,} \qquad \frac{\partial P}{\partial z} \cong 0 \tag{6}$$

The required initial and boundary conditions are

$$t = t_{\text{in}}, \qquad V_r = f_1, \qquad V_\theta = f_2, \qquad h_z = f_3$$

$$r = 0, \qquad V_r = V_\theta = 0, \qquad \frac{\partial h_z}{\partial r} = 0$$

$$r \to \infty, \qquad rV_\theta \to 1$$

The steady-state subsets of Eqs. (1), (3), (5), and (6) are the same formulations that have been used before to produce most of the classical vortices models, such as Oseen–Lamb, Taylor, Burgers, Sullivan, and several others. If the radial and axial velocity components are not negligible, then the preceding system of equations is underdetermined because there are more unknown quantities than available equations. In this case, one of the dependent variables must be preassumed, and the rest are then obtained analytically. The last statement implies that there are infinite velocity and pressure profiles that satisfy these equations. This is the reason behind the emergence of a multitude of past vortex solutions. For instance, in Burgers's[6] solution, a linear variation of the radial velocity was presupposed. In the n family of vortices, the shape of the tangential velocity was preassumed and the other dependent variables were determined analytically. Novel similarity variables are derived and have been used to transform the partial differential equation into ordinary sets. Elementary dimensional analysis suggests that $r_{\max} = \text{const}\sqrt{t} \to r/r_{\max} = r/\text{const}\sqrt{t}$. Consider the following similarity variables: $\eta = r/\sqrt{t},\ V(\eta) = V_\theta(t,r)t^L$, and $U(\eta) = V_r(t,r)Ret^M$, where L and M are unknown constants to be evaluated from substituting back into the governing equations. The tangential momentum equation gives $L + M + \frac{1}{2} = L + 1 \to M = \frac{1}{2}$ no matter the value of (L). The preceding relationship points out that the tangential momentum equation will be homogenized for any (L) value. Here, however, we will assume that $L = \frac{1}{2}$ because, for this case, several solutions to the momentum equation are already known. When the mass conservation equation is used, an expression for the axial velocity is also found. Similarly, one can proceed using the radial momentum equation and find the pressure variation as well. In summary, we consider here the following variable transformation relations:

$$\eta = r\big/\sqrt{t}, \qquad V(\eta) = V_\theta(t,r)\sqrt{t},$$

$$U(\eta) = V_r(t,r)Re\sqrt{t} - \eta/2$$

$$H(\eta) = 1 + h_z(t,r)Ret, \qquad \Delta\Pi(\eta) = tP(t,r) \tag{7}$$

Therefore, the partial differential equations (1), (5), and (3) convert into the following ordinary set in sequence:

$$\eta^{-1}[U\eta]' + H(\eta) = 0 \tag{8}$$

$$\eta^{-1}V^2 = \Delta\Pi' \tag{9}$$

$$V'' + \eta^{-1}(1 - U\eta)V' - \eta^{-2}(1 + U\eta)V = 0 \tag{10}$$

subject to the boundary conditions

$$\eta = 0, \qquad U = V = 0, \qquad H' = 0$$

$$\eta \to \infty, \qquad \eta V \to 1$$

The initial conditions will automatically satisfied if the velocity and pressure at the start assume their respective distributions at time level $t = t_{\text{initial}} \neq 0$.

The steady-state subset is then

$$r^{-1}[rV_r]' + h_z = 0 \tag{11}$$

$$r^{-1}V_\theta^2 = \Delta P' \tag{12}$$

$$V_\theta'' + r^{-1}(1 - rV_r)V_\theta' - r^{-2}(1 + rV)V_\theta = 0 \tag{13}$$

respecting the boundary conditions

$$r = 0, \qquad V_r = V_\theta = 0, \qquad h_z' = 0$$

$$r \to \infty, \qquad rV_\theta \to 1$$

In this case, the primes represent differentiation with respect to r. It is clear that the two groups given by Eqs. (8–10) and (11–13) are dual, which further indicates that if a steady vortex is known, then the corresponding decaying version can be obtained through a straightforward variable transformation and vice versa. Apparently, other flow properties such as vorticity also enjoy the similarity. Concentrated vortices are those at which most of the vorticity dwells within the core. Therefore, the percentage of vorticity within the core stays the same during the vortex decay. Thus, if a vortex is concentrated at the early decay, it will retain the property during its entire diffusion process.

Table 1 Approximate Reynolds number values for some typical vortices

Type of vortex	r_c, m	$V_{\theta\,\max}$, m/s	$Re \sim$	$\delta \sim$
Tornadoes	10	60	10^8	10^{-8}
Dust devils	3	10	10^6	10^{-6}
Whirlpools	15	5	10^8	10^{-8}
Cyclone chamber	0.05	20	10^5	10^{-5}
Bathtub	0.02	0.1	10^4	10^{-4}
Aerodynamic	1	10	10^5	10^{-5}

III. Results and Discussion

A. Transformation of Steady Vortices into Time Decay

1. n Family

In a recent Technical Note,[13] the similarity transformation was illustrated and validated using a single experimental data set from the literature. The family member, $n = 2$ vortex model has been selected to test the methodology for simplicity. Here we extend the approach to propose the general decay expressions for the entire n family of steady vortices. Based on the present theoretical space–time duality, the radial, tangential, and axial velocity components as a function of space and time must then be given by

$$V_r(t,r)Re = \frac{1}{2}\left[\frac{r}{t} - \frac{4(n+1)r^{2n-1}}{(t^n + r^{2n})}\right], \quad V_\theta(t,r) = \frac{r}{(t^n + r^{2n})^{1/n}}$$

$$V_z(t,r)Re = \left[\frac{4nt^n(n+1)r^{2(n-1)}}{(t^n + r^{2n})^2} - \frac{1}{t}\right] \qquad (14)$$

At the limit, as (t) goes to zero, the velocity components tend to

$$\lim_{t\to 0} V_\theta = 1/r, \qquad \lim_{t\to 0} V_r = \lim_{t\to 0} V_z = \pm\infty$$

This model is, therefore, the offspring of a decaying vortex, originally potentially with singular radial and axial velocities. Obviously, such a beginning of the fluid properties renders the continuum postulate questionable, and one should only apply it for time levels considerably larger than the molecular mean free time, for example, t_{in}. However, when $t = 1$, the tangential velocity arrives at the

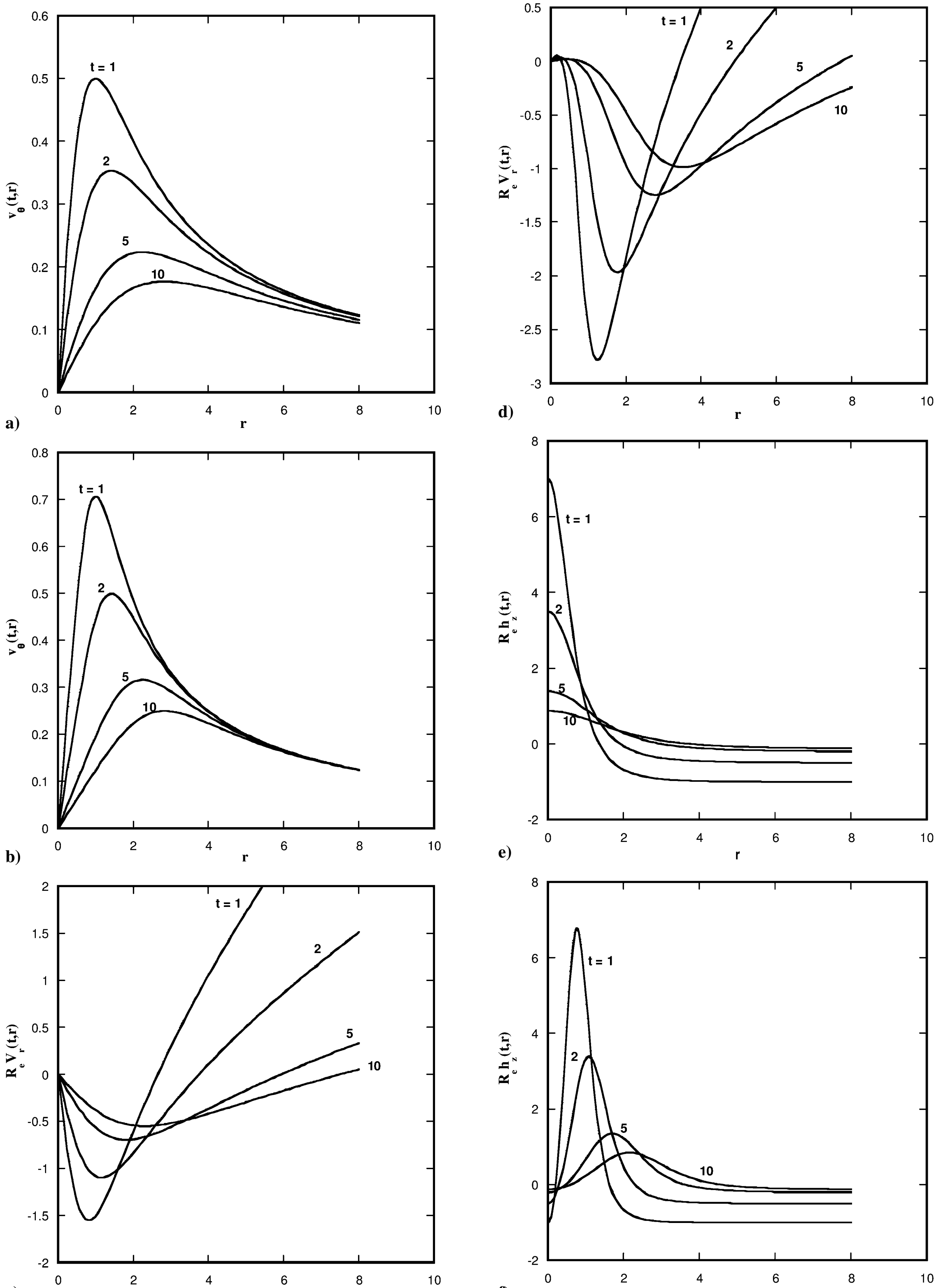
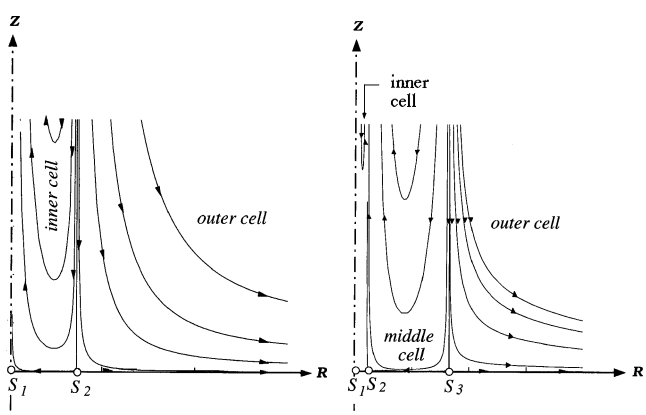


Fig. 1 Chronological distribution of velocity components during the decaying of n = 1 and n = 2 vortices: a) swirl, n = 1; b) swirl, n = 2; c) radial, n = 1; d) radial, n = 2; e) axial, n = 1; and f) axial, n = 2.

n profile. Therefore, in the present study, we will assume that the vortex decay is gradually starting from the steady tangential velocity profile when $t = 1$. This way, the equations will automatically satisfy the initial conditions. Among the n members, $n = 1$ (which is also known as Scully's[14] vortex) and $n = 2$ have been used by several researchers in various studies in the past. In addition to the fact that they are known to provide reasonable approximations to real problems, the pressure in both is given explicitly. For these reasons, our subsequent discussions will concentrate on the last two. Chronological profiles for the tangential velocity as a function of the radius for $n = 1$ and 2 members of the n family are given in Figs. 1a and 1b. The swirl velocity is clearly seen to diminish for both members with time, approaching asymptotically zero as t assumes very large values. The radial and axial velocity profiles for $n = 1$ and 2 members are in Figs. 1c and 1d and 1e and 1f, respectively. Figures 1b and 1c show that for both $n = 1$ and $n = 2$ vortices, far from the origin, the radial velocity is asymptotic linear, $V_r(t, r)Re = r/2t$, whereas the axial velocity approaches the constant value $h_z(t, r)Re = -1/t$.

The streamline patterns generated in the meridional plane (Fig. 2) indicate that, for the $n = 1$ member, a two-celled vortex is present, whereas a three-celled vortex arises for $n = 2$. In both cases, the fluid in the outer cell diverges. We have shown that steady vortices can be transformed into time decaying and vice versa and that this transformation depends solely on the similarity variable η. In other words, if η is indeed the variable that performs this transformation, then both, steady and decaying, vortices, irrespective of their size, strength, and time level, should collapse into a single curve. The observed tangential velocities from various experiments are in Fig. 3 (Refs. 15–19). Figure 3 shows that all of the data collapsed reasonably into the theoretical profile. It is indeed remarkable that such a correlation was attained in spite of the difficulties encountered with the experimental technique, which are mainly associated with the accurate location of the vortex center from the velocity measurements and/or the unsteady character of the vortex core due to the existence of various forms of instability waves. Additionally, although in these experiments the tangential velocity is in fact larger that the other two components, for example, as reported by Yan et al.,[17] it may not necessarily be several orders of magnitude greater. The last suggests that, in reality, the analogy might be preserved under less restrictive conditions than those imposed by the present simple theory. This particular character of the azimuthally flow properties such as the tangential velocity, vorticity, circulation, and pressure is also retained by vortices with substantially different meridional flow such as the trailing vortices in the Newman–Batchelor flow regime. This is the reason behind the use of the general tangential velocity form to curve fit the experimental data of helicopter blade tip vortices.[20]



a) n = 1 two cells **b) n = 2 three cells**

Fig. 2 Streamline patterns at t = 1, for a) n = 1 and b) n = 2 members of the algebraic n-vortex model.

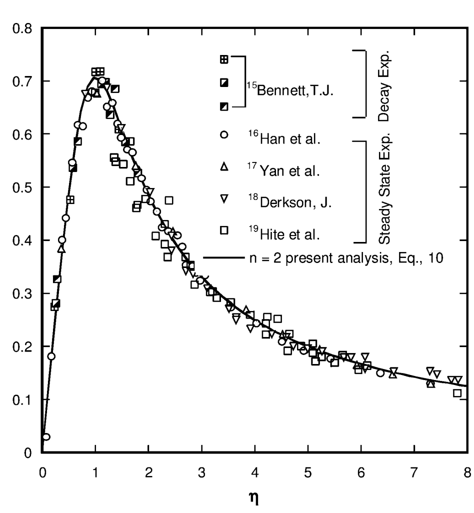


Fig. 3 Tangential velocity collapse for various sets of steady and decaying eddies.

2. *Sullivan's Steady Vortex*

A double-cell steady vortex proposed by Sullivan,[8] that is, within the inner cell there is a central downdraft in the axial direction and a divergence of the flow in the radial direction. These properties are displayed by negative and positive values of the axial and radial velocity components in the inner cell, respectively. The opposite effect is evident in the outer cell. The transformed, time-dependent variables for the Sullivan's vortex are

$$V_r(t, r)Re = -\left(\tfrac{3}{2}\right)(\sigma r/t) + (6/\sigma r)[1 - \exp(-\sigma^2 r^2/t)]$$

$$V_\theta(t, r)\sigma = (1/r)[H(t, r)/H(\infty)]$$

$$h_z(t, r)Re = (3\sigma/t)[1 - 4\exp(-\sigma^2 r^2/t)] \tag{15}$$

where

$$H(t, r) = \int_0^{\sigma^2 r^2/t} \exp\left[-\lambda + 3\int_0^{\lambda}\left(\frac{1 - e^{-s}}{s}\right) \mathrm{d}s\right] \mathrm{d}\lambda$$

$$H(\infty) = \lim_{L\to\infty} \int_0^{L} \exp\left[-\lambda + 3\int_0^{\lambda}\left(\frac{1 - e^{-s}}{s}\right) \mathrm{d}s\right] \mathrm{d}\lambda$$

$$= 37.904514$$

The constant $\sigma = 2.4976$ is used to make the velocity maximum at $r = 1$, when $t = 1$. The velocity profiles for the transformed Sullivan model are shown in Fig. 4. The tangential velocity of this model displays a similarity with that of Fig. 4a. However, the meridional flow corresponding to Figs. 4b and 4c is three-celled one, whereas the time-dependent Sullivan vortex retains its original two-cell character. Furthermore, in the outer cell the flow spirals inward for the latter, whereas in the former it spirals outward.

B. Transformation of Time-Decaying Vortices into Steady Counterpart

A few models in the literature can describe the time decay of vortices. For instance, Taylor,[5] Oseen,[2] Bellamy-Knights,[11] and Aboelkassem et al.[21] Both Taylor and Oseen are single-cell vortices and are possible exact solutions to the same set of equations. A general time-decay model that can vary from Oseen-like to Taylor-like vortices is recently given by Aboelkassem et al.,[21] which is an exact solution to Navier–Stokes equations in the absence of meridional flow. In the preceding section, we have shown that using the transformation a steady vortex can be converted into a time decaying vortex. The process could also be reversed, that is, given a decaying vortex the homologous steady vortex can be derived. The Bellamy-Knights[11] family of vortices that decay in a nonzero meridional flow will be used to show the transformation. Based on Sullivan's work, Bellamy-Knights derived a set of decaying vortices with the following tangential velocity distributions:

$$V_\theta(t, r) = (1/r)[H_\zeta(t, r)/H_\zeta(\infty)] \tag{16}$$

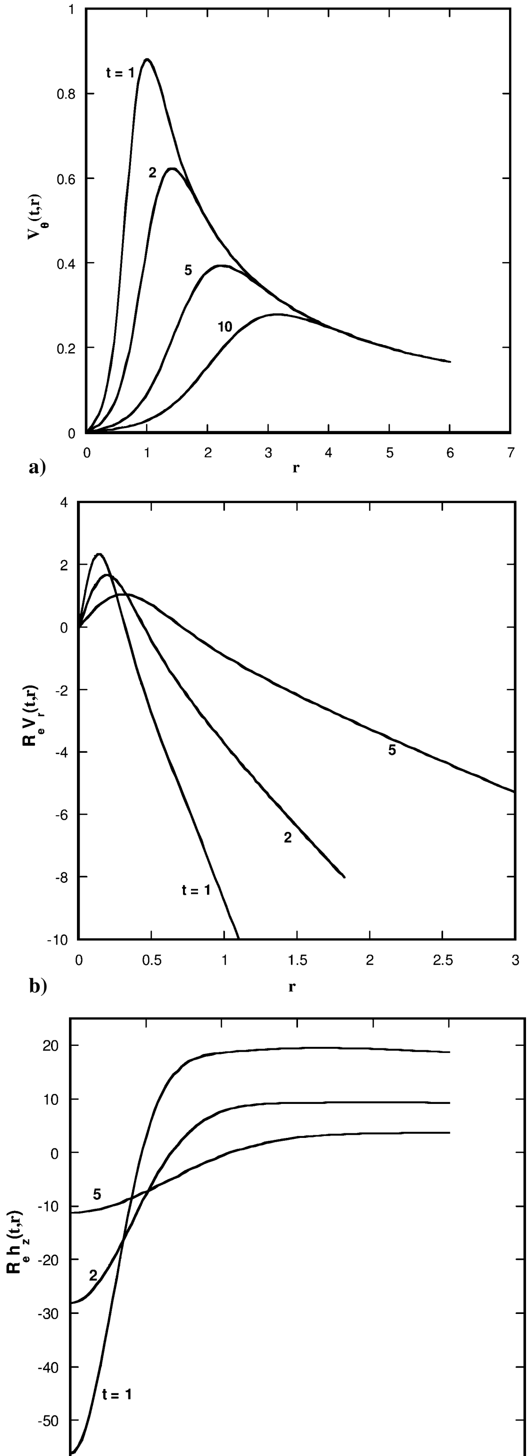


Fig. 4 Velocity components decay for the steady Sullivan diffusing vortex.

where

$$H_k(t,r) = \int_0^{\sigma_k[3r^2/4t(3-k)]} \exp\left\{-\lambda + k\int_0^{\lambda}\left(\frac{1-e^{-s}}{s}\right)\mathrm{d}s\right\}\mathrm{d}\lambda$$

$$H_k(\infty) = \lim_{L\to\infty}\int_0^{L} \exp\left\{-\lambda + k\int_0^{\lambda}\left(\frac{1-e^{-s}}{s}\right)\mathrm{d}s\right\}\mathrm{d}\lambda$$

Transformation of the this velocity expression into steady state yields

$$V_\theta(r)\sigma_k = (1/r)[H_k(r)/H_k(\infty)] \tag{17}$$

where

$$H_k(r) = \int_0^{\sigma_k[3r^2/4(3-k)]} \exp\left\{-\lambda + k\int_0^{\lambda}\left(\frac{1-e^{-s}}{s}\right)\mathrm{d}s\right\}\mathrm{d}\lambda$$

Parameter σ_κ is once again used to make the extreme velocity for $t = 1$ value happen at $r = 1$, where k takes values that are less than 3. Meanwhile, the conversion yields the rest of the velocity components,

$$V_r(r)Re = 2\Big((k/\sigma_k r)\big\{1-\exp\big[-3\sigma_k^2 r^2\big/4(3-k)\big]\big\} \\ -3\sigma_k r/4(3-k)\Big) \tag{18}$$

$$h_z(r)Re = [k/(3-k)]\big\{1-3\exp\big[-3\sigma_k^2 r^2\big/4(3-k)\big]\big\}+1 \tag{19}$$

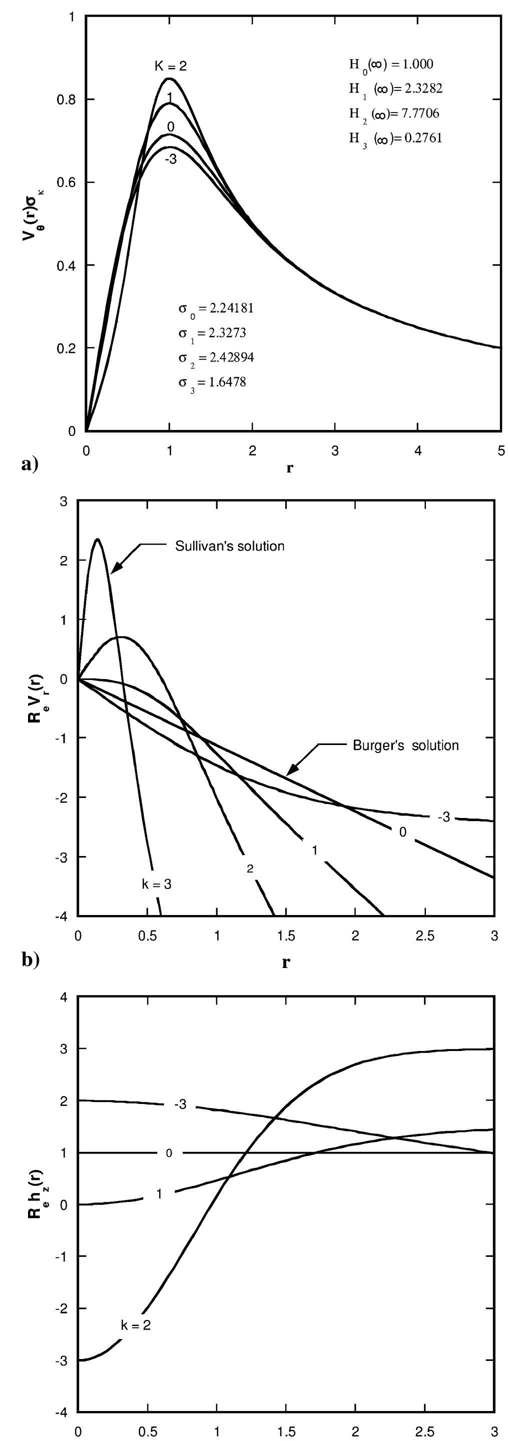


Fig. 5 Bellamy-Knights steady velocity components recovered using the present theory.

Different choice of values for σ_k will give rise to a family of steady vortex profiles. Bellamy-Knights[11] extended the solution of Sullivan to yield an unsteady two-cell family of solutions. Here we will go in reverse, that is, recover the steady-state cases that have produced Bellamy-Knights's vortices. A sample of tangential velocity distributions is shown in Fig. 5a. When $k = 0$, the derived steady-state Bellamy-Knights azimuthal velocity distribution is the same as Burgers's. For k values less than or equal to 1, the meridional flow consists of one cell, and it is converging (Figs. 5b and 5c). Farther than $k = 1$, the flow transforms into a two-cell vortex with the tangential velocity approaching asymptotically Sullivan's profile as $k \to 3$. There is no solution for this formulation beyond $k = 3$. Therefore, the present topology provides steady solutions for Sullivan-like vortices that are different from the original Sullivan vortex.

IV. Conclusions

The theoretical development presented here provides a space–time duality for self-similar intense vortices. The methodology was first derived and then its validity was tested using several known classical vortices. In this Note, we have examined the similarity properties of a group of intense vortices. The system of equations that describes the general problem was condensed using a similarity variable that combines space and time. The revealed duality among the vortex decay and the steady-state subset made possible the transformation of various known vortex models from decaying to steady state and vice versa. Vortices of this type were found to remain concentrated and intense throughout the decay phase. In the presence of a meridional flow, the simplified problem is underdetermined, which explained the reason behind the emergence of a multitude of past solutions. The main characteristics of the analogy were justified by the data of various experimental studies.

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